2103000203023001/2111000303023001
EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (THIRD SEMESTER)
MTH-301 MATHEMATICS PAPER - V

[Time: As Per Schedule] **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **BACHELOR OF SCIENCE (THIRD SEMESTER)**
   b. Name of the Subject : **MTH-301 MATHEMATICS PAPER - V**
      Subject Code No : **2103000203023001/2111000303023001**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual symbols.

Seat No:

Student's Signature

Q.1 Answer any five questions. **10**

(1) Define: Limit of a bivariate function.

(2) Find the limit of the function: $\lim_{(x,y)\to(1,2)}(xy-3x+4)$

(3) Find U_y, U_z for the function $U=\frac{1}{3}xyz+log\frac{yz}{x^2}$

(4) Find the degree of the function $f(x,y)=\frac{x^3-y^3}{\sqrt{x}-\sqrt{y}}$

(5) Write the Taylor series for a bivariate function

(6) State sufficient conditions for an extreme point of a bivariate function.

(7) If $\vec{r}=x\hat{\imath}+yj+zk$ then prove that $div\,\vec{r}=3$

(8) Prove that $\nabla\times\nabla f=\vec{0}$.

Q.2 Answer any five questions. 10

(1) If $z = f(x,y)$ where z is a differentiable function of x and y and the function $\phi: t \to \phi(t) = x$ and $\Psi: t \to \Psi(t) = y$ are the differentiable functions of t then $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$.

(2) Discuss the continuity of the function $f(x,y)$ at the point (0,0) where

$$f(x,y) = \frac{x\sin(x^2+y^2)}{x^2+y^2};\ (x,y) \neq (0,0)$$
$$= 0 \quad ;\ (x,y) = (0,0)$$

(3) If $z(x+y) = x^2 + y^2$ then prove that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$

Q.3 Answer any two questions. 10

(1) If f is a differentiable homogeneous function in x, y of degree m then prove that $xf_x + yf_y = mf(x,y)$.

(2) Expand $f(x,y) = \log(xy)$ in the powers of $(x-1)$ and $(y-1)$.

(3) If $u_1 = \frac{x_2x_3}{x_1}, u_2 = \frac{x_1x_3}{x_2}, u_3 = \frac{x_1x_2}{x_3}$ then prove that $J(u_1, u_2, u_3) = 4$.

Q.4 Answer any two questions. 10

(1) State and prove necessary condition for a minimum point of a bivariate function.

(2) Find maximum and minimum value of the function.
$f(x,y) = 2(x-y)^2 - x^4 - y^4$

(3) Discuss about the extreme points of the function $u = x^3y^2(1-x-y)$.

Q.5 Answer any two questions. 10

(1) If $\vec{f} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ then find $(i) curl\ \vec{f}$ $(ii) curl\ curl\ \vec{f}$.

(2) State the condition for the vector to be solenoidal and show that $\frac{\vec{r}}{r^3}$ is solenoidal where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}\ and\ r = |\vec{r}|$

(3) If c is the triangle formed by vertices (1,0), (0,1) and (-1,0) then find $\int_c (y^2dx - x^2dy)$.

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